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AQA

ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics 6360

MPC4 Pure Core 4

Mark Scheme

2006 examination – June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Key To Mark Scheme And Abbreviations Used In Marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

$\surd$or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MPC4

Q	Solution	Marks	Total	Comments
1 (a)(i)	$p(2)=0$	B1	1	
(ii)	See $-\frac{1}{2}$	B1		
	$p\left(-\frac{1}{2}\right)=6\times\left(-\frac{1}{8}\right)-19\times\frac{1}{4}+9\left(-\frac{1}{2}\right)+10$	M1		Use $\pm\frac{1}{2}$
	$=0$	A1	3	Arithmetic to show = 0 and conclusion. Long division : 0/3
(iii)		B1		$x-2$
	$p(x)=(2x+1)(x-2)(3x-5)$	B1	2	Complete expression
(b)	$\frac{3x(x-2)}{(2x+1)(x-2)(3x-5)}$	M1		For $\frac{3x(x-2)}{\text{their (a)(iii)}}$
	$=\frac{3x}{(2x+1)(3x-5)}$	A1	2	Or $\frac{3x}{6x^2-7x-5}$ No ISW on A1
	Total		**8**	
2(a)	$(1-x)^{-3}=1+(-3)(-x)+\frac{(-3)(-4)(-x)^2}{2}$	M1		$1\pm3x+x^2$ term
	$=1+3x+6x^2$	A1	2	
(b)	$\left(1-\frac{5}{2}x\right)^{-3}=1+3\left(\frac{5}{2}x\right)+6\left(\frac{5}{2}x\right)^2$	M1		$x\to\frac{5}{2}x$, incl. $\left(\frac{5}{2}x\right)^2$ seen or implied (or start again)
	$=1+\frac{15}{2}x+\frac{75}{2}x^2$	A1	2	CAO OE
………… ………… ………… **(c)**	$\left\lvert\frac{5}{2}x\right\rvert<1 \quad \lvert x\rvert<\frac{2}{5}$	M1A1	2	Sight of $\frac{\pm5}{2}$ or $\frac{\pm2}{5}$
(d)	$=8(1+\frac{15}{2}x+\frac{75}{2}x^2)=8+60x+300x^2$	M1		$k\times$ their $\left(1-\frac{5}{2}x\right)^{-3}$
		A1F	2	ft only on $8\left(1-\frac{5}{2}x\right)^{-3}$
	Alternatively, start again:			
	$8\times$ expression or $k\times\left(1-3\left(\pm\frac{5}{2}x\right)\right)$	(M1)		
	CAO	(A1)		
	Total		**8**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$9x^2-6x+5$			
	$=3(3x-1)(x-1)+A(x-1)+B(3x-1)$	B1		Or $3+\dfrac{6x+2}{(3x-1)(x-1)}$
	$x=1 \qquad x=\dfrac{1}{3}$	M1		Substitute $x=1$ or $x=\dfrac{1}{3}$
	$B=4 \qquad A=-6$	A1A1	4	Or equivalent method (equating coefficients, simultaneous equations)
(b)	$\int=\int 3-\dfrac{6}{3x-1}+\dfrac{4}{x-1}\,\mathrm{d}x$	M1		Attempt to use partial fractions
	$=3x\ldots$	B1		
	$-2\ln(3x-1)+4\ln(x-1)(+c)$	M1		$p\ln(3x-1)+q\ln(x-1)$ Condone missing brackets
		A1F	4	Follow through on *A* and *B;* brackets needed.
	Total		**8**	
4(a)(i)	$\sin 2x=2\sin x\cos x$	B1	1	
(ii)	$\cos 2x=2\cos^2 x-1$	B1	1	
(b)	$\sin 2x-\tan x=2\sin x\cos x-\dfrac{\sin x}{\cos x}$	M1		Use of their $\cos 2x$ or $\sin 2x$
		M1		Use of $\tan x=\dfrac{\sin x}{\cos x}$ and the other double angle identity
	$=\sin x\left(2\cos x-\dfrac{1}{\cos x}\right)$			
	$=\sin x\left(\dfrac{2\cos^2 x-1}{\cos x}\right)=\tan x\cos 2x$	A1	3	AG convincingly obtained
(c)	$\tan x\cos 2x=0 \qquad x=180$	B1		Ignore $x=0$, $x=360°$ & any others outside range
	$\cos 2x=0$ or $\cos^2 x=\dfrac{1}{2}$ $\left(\text{or } \sin^2 x=\dfrac{1}{2}\right)$	M1		
	$x=45$	A1		
	$x=135, 225, 315$	A1	4	CAO max 3/4 for answers in radians
	Total		**9**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$x=1 \quad y^2-y+3-5=0$	M1		
	$(y-2)(y+1)=0$	M1		Attempt to solve quadratic equation with $x=1$
	$y=2 \quad y=-1$	A1	3	
(b)(i)	$2y\frac{dy}{dx}-x\frac{dy}{dx}-y+6x=0$	B1B1		$+6x$; $-5 \to 0$
		B1		Chain rule
		M1A1		Product rule (M1 two terms)
	$6x-y+(2y-x)\frac{dy}{dx}=0$	A1	6	Factorise and obtain answer given
	Alternative			
	$\frac{dy}{dx}(y-x)^2=(y-x)(0-6x)$	(B1)		$5 \to 0$
		(B1)		$-6x$
	$-(5-3x^2)\left(\frac{dy}{dx}-1\right)$	(M1)		Recognisable attempt at quotient rule
		(A1)		Completely correct OE
	$\frac{dy}{dx}\left[(y+x)^2+(5-3x^2)\right]=(y-x)(-6x)$ $+(5-3x^2)$	(A1)		Factorise out $\frac{dy}{dx}$
	Given answer	(A1)		Correct answer from correct working Be convinced
(ii)	$(1,2) \quad \frac{dy}{dx}=-\frac{4}{3}$	M1		Substitute $x=1$ and one y value from (a)
	$(1,-1) \quad \frac{dy}{dx}=\frac{7}{3}$	A1F	2	Both; follow on candidates y s OE $\frac{-7}{-3}$; 3SF
(iii)	$y-6x=0$	B1		
	$(6x)^2-x\times 6x+3x^2-5=0$	M1		
	$36x^2-6x^2+3x^2-5=0$			
	$33x^2-5=0$	A1	3	AG convincingly obtained
	Total		**14**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
6(a)(i)	$\overrightarrow{OC} = 2\begin{bmatrix}3\\2\\-1\end{bmatrix} = \begin{bmatrix}6\\4\\-2\end{bmatrix}$	B1	1	(Penalise coordinates once only)
(ii)	$\overrightarrow{AB} = \begin{bmatrix}3\\2\\-1\end{bmatrix} - \begin{bmatrix}2\\4\\1\end{bmatrix} = \begin{bmatrix}1\\-2\\-2\end{bmatrix}$	M1 A1	 2	$\overrightarrow{OA} - \overrightarrow{OB}$ or $\overrightarrow{OB} - \overrightarrow{OA}$ or 2/3 correct cpts. A0 for line AB
(b)(i)	$AC^2 = (6-2)^2 + (4-4)^2 + (-1-2)^2 = 25$ $AC = 5$	M1 A1	 2	|Components of AC| AG
(ii)	$\overrightarrow{AB} \bullet \overrightarrow{AC} = \begin{bmatrix}1\\-2\\-2\end{bmatrix} \bullet \begin{bmatrix}4\\0\\-3\end{bmatrix} = 4 + 6 = 10$	M1 A1F		Clear attempt to use $\overrightarrow{AB}$ and $\overrightarrow{AC}$ ft $\overrightarrow{AB}$ from a(ii) and/or $\overrightarrow{AC}$ from b(i)
	$3 \times 5 \times \cos\theta = 10$	M1		Use of $\|a\|\ \|b\|\cos\theta = \mathbf{a.b}$ with one correct $\|\ \|$ and **a.b** evaluated
	$\theta = 48.189 \approx 48\,°$	A1	4	CAO (AWRT)
	Alternative: use of cos rule Find 3rd side + use cos rule	(M2) (A1F) (A1)		 ft on previously found vectors CAO (AWRT)
(c)	$\overrightarrow{BP} = \begin{bmatrix}\alpha - 3\\ \beta - 2\\ \gamma - -1\end{bmatrix}$	B1		
	$\begin{bmatrix}4\\0\\-3\end{bmatrix} \bullet \overrightarrow{BP} = 0$	M1		Their $\overrightarrow{BP}$
	$4\alpha - 3\gamma - 15 = 0$	A1	3	AG convincingly obtained
	Total		**12**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
7	$\int \frac{\mathrm{d}y}{y^2} = \int 6x \, \mathrm{d}x$	M1		Attempt to separate Either dx or dy in right place
	$-\frac{1}{y} = 3x^2 (+C)$	A1A1		$-\frac{1}{y}$; $3x^2$
	$x = 2 \quad y = 1 \quad C = -13$	M1 A1		Use (2,1) to find a constant. CAO
	$y = \frac{1}{13 - 3x^2}$	A1	6	CAO OE
	Total		**6**	
8(a)(i)	$(5000 - x)$ seen in a product	B1		Could be implied, eg $5000a - xa$
	$\frac{\mathrm{d}x}{\mathrm{d}t} = kx(5000 - x)$	B1	2	
(ii)	$200 = k \times 1000 \times (5000 - 1000)$	M1		$\frac{\mathrm{d}x}{\mathrm{d}t} = 200, x = 1000$ in their diff. equation Condone ts and $t = 0$ for M1
	$k = 0.00005$	A1	2	CAO OE
(b)(i)	$t = 4\ln\left(\frac{4 \times 2500}{5000 - 2500}\right) = 5.5$ (hours)	M1 A1	2	$x \to 2500$ (or 4 ln 4) CAO
(ii)	$\mathrm{e}^{\frac{30}{4}}$	B1		
	$\mathrm{e}^{7.5} = \frac{4x}{5000 - x}$	M1		OE
	$5000 \times \mathrm{e}^{7.5} = x(4 + \mathrm{e}^{7.5})$	m1		Soluble for x
	$x = 4988.96.. \Rightarrow 4989$ rabbits infected	A1	4	Or 4988 or 4990; integer value only
	Total		**10**	
	TOTAL		**75**	